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RELATION BETWEEN IONIZATION YIELD FUNCTION AND IONIZING CAPABILITY DUE TO SOLAR COSMIC RAYS IN THE IONOSPHERE AND STRATOSPHERE CALCULATED BY CORSIKA AND CORIMIA PROGRAMS RESPECTIVELY

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Key words: *cosmic ray ionization effects, ionization yield function, CORSIKA, ionizing capability, CORIMIA, ionization mechanisms, ionosphere, stratosphere*

Abstract: *The cosmic rays produced in solar flares are one of most important manifestation of solar activity. Two approaches have been developed to compute the cosmic ray ionization, e.g.: i) the code CORSIKA, including the package FLUKA, is statistical and he is based on a Monte Carlo simulation of the atmospheric cascades at Extensive Air Showers (EAS), while ii) the model CORIMIA is analytical. Usually the statistical simulations utilize the ionization yield Y function, while the analytical models use the ionizing capability C function. In the present work new formulas for ionization yield functions Y and ionizing capability C are derived for the solar cosmic rays, i.e. for sub-relativistic approximation. It is obtained also the relationship between the functions Y and C, which are practicable for the calculation of atmospheric ionization.*

ВРЪЗКА МЕЖДУ ПОРАЖДАЩАТА ЙОНИЗАЦИЯ ФУНКЦИЯ И ЙОНИЗИРАЩАТА СПОСОБНОСТ НА СЛЪНЧЕВИТЕ КОСМИЧЕСКИ ЛЪЧИ В ЙОНОСФЕРАТА И СТРАТОСФЕРАТА ИЗЧИСЛЕНИ ЧРЕЗ ПРОГРАМИТЕ CORSIKA И CORIMIA РЕСПЕКТИВНО

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Ключови думи: *йонизационни ефекти на космически лъчи, пораждаща йонизацията функция, CORSIKA, йонизираща способност, CORIMIA, йонизационни механизми, йоносфера, стратосфера*

Резюме: *Космическите лъчи генерирани при слънчевите ерупции са едни от най-важните прояви на слънчевата активност. Понастоящем са разработени два подхода за изчисляването на йонизацията от космическите лъчи, т.н.: i) програмата CORSIKA, която включва пакета FLUKA, е статистическа и се базира на симулациите по метода Монте Карло на атмосферни каскади при широките атмосферни порои (ШАЛ), докато ii) моделът CORIMIA е аналитичен. Обикновено статистическите методи използват пораждащата йонизация функция Y, докато аналитичните модели използват функцията на йонизиращата способност C. В настоящата работа са изведени нови формули за пораждащата йонизация функция Y и за йонизиращата способност C от слънчеви космически лъчи, т.е. използвана е субрелативистична апроксимация. Получена е също така връзката между функциите Y и C, която има практическо значение за изчисляването на атмосферната йонизация.*

Introduction

The cosmic rays (CR) produced in solar flares (and in some other high-energy solar processes) are one of most important manifestation of solar activity and one of the main agents in solar-terrestrial relations [1, 2, 3]. Two approaches have been developed to compute the CR ionization, e.g.: *i*) the code CORSIKA (COsmic Ray SImulations for KAscade), including the Monte

Carlo package FLUKA (FLUktuierende KAskade) is statistical and he is based on a Monte Carlo simulation of the atmospheric cascade [4, 5], while *ii*) the model CORIMIA (COsmic Ray Ionization Model for Ionosphere and Atmosphere) is analytical [6]. Usually the statistical simulations utilize the ionization yield *Y* function [4, 5], while the analytical models use the ionizing capability *C* function [1, 7]. In the present work new formulas for ionization yield functions *Y* and ionizing capability *C* are derived for the solar cosmic rays and anomalous cosmic rays, i.e. for sub-relativistic approximation.

Statistical approach in ionization models

In *the statistical approach* is calculated initially the ionization yield function *Y* [4, 5]. Then, the integration by energy is carried out. In this approach all results depend on exact atmospheric density profile. In order to minimize this uncertainty, the CR ionization is computed per gram of the atmospheric matter rather than per cm^3. In this case, the uncertainties of the *q(h)*, computed using different atmospheric density profiles, do not exceed 1–2% in the low troposphere, which is less than statistical errors of computations.

CORSIKA is a physics computer software for simulation of extensive air showers induced by high energy cosmic rays [8]. It may be used up to and beyond the highest energies of 100 GeV. Program utilizes QGSJET (Quark Gluon String with JETs), and DPMJET (Dual Parton Model with JETs), and VENUS models, which are based on the Gribov-Regge theory and SIBYLL based on a minijet model. Hadronic interactions at lower energies are described either by the FLUKA (a German acronym for Fluctuating Cascade) model, by the UrQMD (Ultrarelativistic Quantum Molecular Dynamics) model, or by the GHEISHA (Gamma-Hadron-Electron-Interaction SH(A)ower) module [9].

Ionization Yield Function

First we have shown the ionization yield function *Y* which gives the number of ion pairs produced in one gram of the ambient air at a given atmospheric depth by one nucleon of the primary cosmic ray particle with the given energy per nucleon. The ionization yield function *Y* (ion pairs sr cm^2 g^{-1}), which corresponds to the atmospheric depth $\tilde{h}$, the primary cosmic rays with the kinetic energy E_k and the unit unidirectional flux (particles sr^{-1} cm^{-2}), is [4, 5]:

(1) $$Y\left(E_k, \tilde{h}\right) = \frac{2\pi}{Q}\frac{\Delta E}{\Delta \tilde{h}}$$

where ΔE is the mean energy losses in the atmospheric layer centered at the atmospheric depth $\tilde{h}$ per one simulated primary nucleon with the kinetic energy E_k, and Q = 35eV is the average energy needed to produce one ion pair [1], and 2π is the geometrical normalizing factor.

Analytical approach in ionization models

At *the analytical approach* (f.e. the model CORIMIA) the electron production rate *q* [$cm^{-3}s^{-1}$] at height *h* [km] due to CR is [1, 6]:

(2) $$q(h) = \frac{2\pi}{Q}\int_{E_C}^{\infty} D(E)\left(\frac{dE}{dh}\right)dE$$

where *E* is the full energy of the penetrating particles;
$D(E) = K\,E^{-\gamma}$ is their differential spectrum [cm^{-2} $s^{-1}sr^{-1}GeV^{-1}$], *K* and *γ* are constants;
(*dE/dh*) are the ionization losses of CR particles according to the Bohr-Bethe-Bloch formula;
the multiplier 2π shows that the galactic CR penetrate isotropic from the upper hemisphere;
E_c is the corresponding geomagnetic cutoff energy in GeV/nucl.

Ionizing capability

$q(h)$ is closely connected with the CR ionizing capability. In general the ionizing capability function *C*(*h*) at given altitude *h* with atmospheric density *ρ* is determined by the expression [1, 7]:

(3) $$C(h) = q(h)\,/\,\rho(h)$$

This function physically represents the number of electron-ion pairs produced in one gram of matter (in this case atmospheric air) per second and characterizes the ionization effectiveness of the radiation factor [1, 7].

Relation between ionization yield function and ionizing capability

We want to combine the mentioned two approaches. For this purpose we introduce the function *Y* [4, 5] in the expression (2) after which we derive the formula:

(4) $$q(h)=\int_{E_C}^{\infty} D(E)Y(E,h)\rho(h)dE$$

$\rho(h)$ is the density [g cm^{-3}] of atmosphere at height *h* and the ionization yield function *Y* [electron-ion pairs sr cm^2 g^{-1}] can be represented as:

(5) $$Y(E,h)=\frac{2\pi}{Q}\left(\frac{1}{\rho}\frac{dE}{dh}\right)$$

Thus, the wanted connection between the functions $C(h)$ and $Y(h)$ will be:

(6) $$C(>E_C)=\int_{E_C}^{\infty} D(E)Y(E,h)dE$$

Actually the CR ionizing capability $C(h)$ depends on the function of ionization losses and ionization potential of the medium *Q*, on the type of CR nuclei (or group of nuclei: p, He, L, M, H, VH and SH), on their spectrum and geomagnetic threshold of rigidity.

Formula for ionization yield function

Using the relation (5) and the formulas for ionization losses in [1] we can write the following formula for the ionization yield function:

(7) $$Y(E)=5.4\times10^{-2}\frac{Z^2}{\beta^2}\left(\ln\frac{E}{E_0}\beta+4.5-\frac{\beta^2}{2}\right)$$

where *Z* is the charge of the penetrating CR particle,

$\beta = v/c$ is the relative velocity, where *v* is velocity of the particle and *c* is the light velocity;
E and E_0 are the full energy and rest energy of the CR nuclei.

Relativistic approximation

Since the galactic CR are with relativistic energies then the condition

(8) $$\beta = v/c \approx 1$$

(exactly fulfilled at lower and middle latitudes) is valid. After assuming this ratio from the expression of ionization yield function (7) follows:

(9) $$Y(E)=5.4\times10^{-2}Z^2\left(\ln\frac{E}{E_0}+4\right)$$

This means that

(10) $$E >> E_0 \ \text{ or } \ E \approx E_k$$

where

(11) $$E_k = E - E_0$$

is the kinetic energy of the particle. If we place ln E_0 = ln 0.938 in (9), we will receive the formula:

$$Y(E) = 5.4 \times 10^{-2}\ Z^2 (\ln E + 4.064)$$

(12)

or

(13) $$Y(E) = 5.4 \times 10^{-2}\ Z^2 (\ln E + 4)$$

if the energy is expressed in units E_0 = 0.938 GeV.

Sub-relativistic approximation

In [3] a simplified formula is derived for the ionizing capability of solar cosmic rays:

(14) $$C(h) = 5.4 \times 10^4 \sum_{\mathrm{i}=1}^{\mathrm{n}} \int_{E_{\mathrm{k\,max}}}^{\infty} \int_{0}^{90^\circ} D_{\mathrm{i}}(E_{\mathrm{k}}) 83 Z_{\mathrm{i}}^2$$

$$\times \frac{\ln\left(E_{\mathrm{k}}^2 - 10^3 \tilde{h} Z_{\mathrm{i}}^2 \sec\theta\right)^{\frac{1}{2}} + 3.2}{\left(E_{\mathrm{k}}^2 - 10^3 \tilde{h} Z_{\mathrm{i}}^2 \sec\theta\right)^{\frac{1}{2}}} \sin\theta\, d\theta\, dE_{\mathrm{k}}$$

as $E_{\mathrm{k\,max}}$ is the boundary energy (MeV/nucl) corresponding to the geomagnetic cutoff rigidity (GV).

If in (14) we ignore the variation of the logarithmic term (it changes much more slowly than the term E_{k}^{-1}), a more simple formula can be derived for the ionizing capability of solar cosmic rays. In the beginning we shall integrate this formula by the angle θ in order to obtain the mean ionization yield function of the isotropic CR particle flux at a given atmospheric depth $\tilde{h}$:

(15) $$Y(\bar{\theta}, \tilde{h}) = 90\ Z^2 \int_0^{\theta_{\mathrm{c}}} \frac{\sin\theta\, d\theta}{\left(E_{\mathrm{k}}^2 - 10^3\ \tilde{h}\ Z^2\ \sec\theta\right)^{1/2}}$$

The solution of this integral is:

(16) $$Y(\bar{\theta}, \tilde{h}) = 90\ Z^2 \left\{ \frac{\left(E_{\mathrm{k}}^2 - E_{\mathrm{A}}^2\right)^{1/2}}{E_{\mathrm{k}}^2} - \frac{\left[E_{\mathrm{k}}^2 - E_{\mathrm{A}}^2(\theta_{\mathrm{c}})\right]^{1/2}}{E_{\mathrm{k}}^2\ \sec\theta_{\mathrm{c}}} + \right.$$
$$+ \frac{1}{2} \frac{E_{\mathrm{A}}^2}{E_{\mathrm{k}}^2} \ln \frac{E_{\mathrm{k}} + \left(E_{\mathrm{k}}^2 - E_{\mathrm{A}}^2\right)^{1/2}}{E_{\mathrm{k}} - \left(E_{\mathrm{k}}^2 - E_{\mathrm{A}}^2\right)^{1/2}} -$$
$$\left. - \frac{1}{2} \frac{E_{\mathrm{A}}^2}{E_{\mathrm{k}}^2} \ln \frac{E_{\mathrm{k}} + \left(E_{\mathrm{k}}^2 - E_{\mathrm{A}}^2\ \sec\theta_{\mathrm{c}}\right)^{1/2}}{E_{\mathrm{k}} - \left(E_{\mathrm{k}}^2 - E_{\mathrm{A}}^2\ \sec\theta_{\mathrm{c}}\right)^{1/2}} \right\}$$

If we introduce the atmospheric cutoff energies

(17) $$E_{\mathrm{A}i} = \left(10^3 \tilde{h} Z_i^2\right)^{1/2} = E_{\mathrm{A}i}(0^\circ)$$

and

(18) $$E_{\mathrm{A}i}(\theta_{\mathrm{c}}) = \left(10^3 \tilde{h} Z_i^2\ \sec\theta_{\mathrm{c}}\right)^{1/2} = E_{\mathrm{A}i}\sqrt{\sec\theta_{\mathrm{c}}}$$

then for the ionizing capability from (14, 16) we obtain:

$$C(h) = 1.8 \times 10^5 \sum_{i=1}^{7} \int_{E_{\mathrm{k\,max}}}^{\infty} D_i(E_\mathrm{k}) \frac{500\, Z_i^2}{E_\mathrm{k}} \times$$

$$\times \left[\left(1 - \frac{E_{\mathrm{A}i}^2}{E_\mathrm{k}^2}\right)^{1/2} - \frac{1}{\sec\theta_\mathrm{c}} \left(1 - \frac{E_{\mathrm{A}i}^2(\theta_\mathrm{c})}{E_\mathrm{k}^2}\right)^{1/2} + \right.$$

(19)

$$+ \frac{1}{2}\left(\frac{E_{\mathrm{A}i}}{E_\mathrm{k}}\right)^2 \left(\ln \frac{1 + \left(1 - E_{\mathrm{A}i}^2 / E_\mathrm{k}^2\right)^{1/2}}{1 - \left(1 - E_{\mathrm{A}i}^2 / E_\mathrm{k}^2\right)^{1/2}} - \right.$$

$$\left. \left. - \ln \frac{1 + \left(1 - E_{\mathrm{A}i}^2(\theta_\mathrm{c}) / E_\mathrm{k}^2\right)^{1/2}}{1 - \left(1 - E_{\mathrm{A}i}^2(\theta_\mathrm{c}) / E_\mathrm{k}^2\right)^{1/2}} \right) \right] dE_\mathrm{k}$$

where $E_{\mathrm{k\,max}}$ is determined by the equations

$$E_{\mathrm{k\,max}} = \max \begin{cases} E_\mathrm{Ri} = 10^3 \left\{ \left[\left(\dfrac{Z_\mathrm{i}}{A_\mathrm{i}}\right)^2 R^2 + 1 \right]^{\frac{1}{2}} - 1 \right\} \\ E_\mathrm{Ai} = \left(10^3 \tilde{h} Z^2 \sec\theta + \varepsilon\right)^{\frac{1}{2}} \\ E_\mathrm{Ei} \end{cases}$$

(20)

i.e. for each altitude E_kmax is the biggest of the three energies: the energy corresponding to the geomagnetic cutoff rigidity E_Ri, to the atmospheric cutoff E_Ai, and to the electric energy cutoff E_Ei outside the geomagnetic field. The energy ε is about 0.1 MeV [2, 9].

In order to estimate this theoretical result a numerical integration must be made. In [1, 3, 6] is considered the biggest SEP event (in the CR history after 1942) from 23.02.1956 and the spectrum

(21) $$D(E_\mathrm{k}) = K\, E_\mathrm{k}^{-n}$$

is used. Precisely with the help of this spectrum should be resolved numerically the integral (19). So shall be obtained the searched values of the ionizing capability of the solar cosmic rays and anomalous cosmic rays, i.e. of the case of sub-relativistic particles, which play an important role in the ionization state of the high and polar latitude ionosphere and stratosphere.

Conclusion

The energetic cosmic rays initiate a nucleonic-electromagnetic cascade in the atmosphere, affecting its physical-chemical properties and ion balance [4]. This is a dominant source of ionization of the lower ionosphere and stratosphere [1, 3, 6, 7]. Our contribution to the models of the CR ionization makes a basis for a quantitative investigation of the mechanisms of solar-terrestrial relationships. CR are a key factor affecting the physical-chemical processes in the stratosphere and lower atmosphere, including the electric conductivities, electric currents and fields [10, 11] and the atmospheric chemistry (e.g. variations and planetary distribution of the ozone [12]). In general CR are a key for understanding the space weather in the system ionosphere - atmosphere and in the Earth's environment.

In the present work we obtained a relation between the two approaches: the determination of the ionization yield *Y* function (used by the statistical code CORSIKA) and ionization capability *C* (used by the analytical model CORIMIA). We found a consistent method for the calculation of atmospheric ionization due to cosmic rays with solar and interplanetary (anomalous CR) origin.

The results of the full Monte Carlo simulation which are tabulated in a form of the ionization yield function [4, 5 etc.] can be applied much more widely. With the help of the achievements of this work the analytical models CORIMIA can use the results of statistical code CORSIKA that will allow comparison between the two approaches. As is known by CORSIKA is possible to estimate the CR ionization below 20-25 km and by the model CORIMIA - above this altitude. So those two approaches are complementary.

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